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ABSTRACT

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Analytical techniques are developed to determine the optimum weight and performance of an ideal landing system designed to attenuate motions such as experienced by a vehicle landing vertically on a planetary surface. Only the use of solid structural materials as energy absorbers are considered. It is shown that the optimum landing system weight can be related to vehicle touchdown velocity and a simple design index. It is intended that the optimum landing system weight as determined by considerations presented in this report serve as a standard for evaluating the relative merit of present and future landing system designs using solid structural materials.

AUTHOR

SYMBOLS

A	=	cross sectional area of energy absorbing material
A_1	=	cross sectional area of energy absorbing material associated with the end of the stroke
$\overline{A}$	=	nondimensional cross sectional area
E_a	=	energy absorbed by the landing system
F	=	force developed by energy absorbing material
g	=	planetary acceleration due to gravity
KE_T	=	vehicle touchdown kinetic energy
$KE_{\dot{g}}$	=	kinetic energy absorbed by onset limiter
KE_g	=	kinetic energy absorbed by deceleration limiter
L	=	total length of the energy absorbing material
m	=	vehicle mass
n	=	deceleration factor in g's
n_o	=	peak deceleration factor
t	=	time after vehicle touchdown
t_1	=	time associated with the end of landing system stroke
V_i	=	vehicle touchdown velocity
W_i	=	internal energy developed by energy absorbing material
W_v	=	vehicle gross weight
W_{LS}	=	landing system weight
W_m	=	weight of landing system energy absorbing material
$\overline{W}_{LS}$	=	*minimum* landing system weight
x	=	vehicle displacement after touchdown as a function of time or landing system stroke, length coordinate of energy dissipating material

$\dot{x}$ = vehicle velocity after touchdown

$\ddot{x}$ = vehicle acceleration after touchdown

$\dddot{x}$ = time rate of change of vehicle acceleration or onset rate

$\bar{\dddot{x}}$ = onset rate in g's/sec

x_g = maximum stroke for deceleration limiter

$x_{\dot{g}}$ = maximum stroke for onset limiter

x_c = maximum stroke for composite energy absorber

$\bar{x}$ = nondimensional stroke

β = $n_o V_i/(x_c \bar{\dddot{x}})$

η = energy absorbing efficiency of landing system

η_m = material efficiency

θ = $x_c \bar{\dddot{x}}/n_o$

λ = $n_o^2 g/(V_i \bar{\dddot{x}})$

ρ = density of energy absorbing material

<u>Subscripts</u>

g = deceleration limiter

$\dot{g}$ = onset limiter

1. INTRODUCTION

Some form of energy dissipation system is necessary to provide a soft landing capability for spacecraft payloads alighting on planetary or lunar surfaces. Inherent in the design of any such system for space applications are stringent requirements on weight, reliability, and storability. Hence, it is readily apparent that an ideal energy absorption system would have minimum weight and volume and maximum reliability.

Weight considerations place emphasis on designs which provide a maximum efficiency in terms of energy absorbed per pound of system. Reliability requirements imply a system characterized by simplicity, predictability, and reproducibility of operation. Also for many applications, maximum storability will correspond to energy absorption systems which employ a minimum working stroke. Since most currently contemplated spacecraft are designed for a single mission capability, the landing system need not be reuseable. Another essential feature is that the system attenuate vehicle motion with a minimum or rebound. Finally, the materials of construction for a landing device obviously must function in a space or planetary environment. This requirement implies a design which has a high tolerance to a vacuum environment, corpuscular and electromagnetic radiation, and possibly temperature extremes.

It has been recognized that landing systems which absorb energy through inelastic deformation of structural materials hold considerable promise for fulfilling the requirements enumerated above. As shown in Ref. (1), the efficiency of these systems not only competes favorably, but in certain cases is superior to other types of systems such as gas bags and retro-rockets. In addition, if the energy dissipating material is designed to deform plastically, the requirement for no vehicle rebound can be satisfied. Furthermore, the stability of many structural materials in a space environment has already been demonstrated.

Because of the varied types of landing systems that can be contemplated, there exists a need to compare energy dissipation systems on a common basis so that their relative efficiency can be established in terms of specified performance requirements. As a consequence, this study is concerned with the establishment of suitable efficiency and design indices for idealized landing systems that utilize structural materials as energy dissipators. The results are obtained in sufficiently general form that they may be applicable to other types of landing systems provided that they conform to the ideal system used in the analysis.

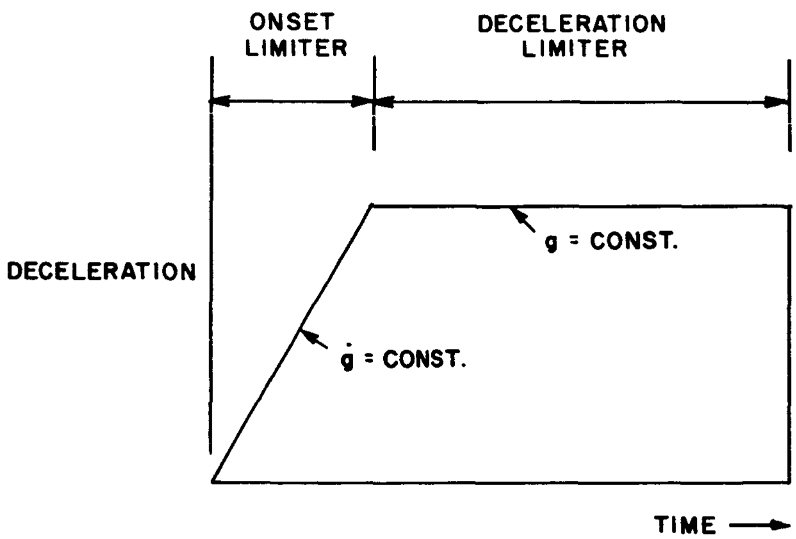
Primary considerations for the design of an energy dissipating system for a planetary landing vehicle include the fragility of the payload, touchdown velocity, and the size and weight of the landing system. Payload fragility can be defined in terms of the allowable peak deceleration and the time rate of change of deceleration (deceleration onset rate or onset rate), whereas, an index of landing system size is the stroke of the energy dissipator required to arrest vehicle landing motions.

The deceleration characteristics of a vehicle employing an ideal energy absorber are assumed to be those shown in Figure 1. As indicated, the vehicle would experience a linear increase in deceleration at a constant onset rate until the allowable peak deceleration is reached. Subsequent vehicle motion occurs at a constant deceleration until the vehicle has been brought to rest. With the deceleration-time signature shown in Figure 1, the landing system arrests vehicle motion consistent with the maximum loads which can be tolerated by the payload. Hence, the associated weight and stroke of the landing system should be a minimum.

It is convenient to consider the ideal energy absorber to be composed of an onset limiter and deceleration limiter with the associated regions of operation indicated in Figure 1. In this manner, the characteristics of each element of the system can be studied separately, and the analysis of the composite system will follow from the integration of its elements. In Section 2 considerations are presented leading to the weight and performance of a deceleration limiter as a landing system. A similar development for an onset limiter as a landing system is given in Section 3. Finally the methods of analysis and results concerning the weight and performance of the composite system shown by Figure 1 are presented in Section 4.

FIG. 1

VEHICLE DECELERATION TIME HISTORY WITH A LANDING SYSTEM EMPLOYING A COMPOSITE ENERGY ABSORBER



2. CONSTANT DECELERATION ENERGY ABSORBER

A salient characteristic of the idealized constant deceleration energy absorber is that the force generated in resistance to vehicle landing motions is constant with time. By Newton's law, the corresponding vehicle deceleration will also be constant as illustrated by Figure 2. (One disadvantage of such a landing system is readily apparent; namely, the rate of change of deceleration is infinite at time zero and "t_1".) Since this type of energy absorption provides control over the magnitude of the peak vehicle deceleration, it is conveniently described as a deceleration limiter.

Stroke Relationships

As shown by Figure 2, the vehicle deceleration-time signature associated with a deceleration limiter energy absorbing element is a rectangular pulse. The corresponding landing system stroke is determined by a double integration of the acceleration time history.

$$x = \iint \ddot{x} \, dt \, dt \tag{1}$$

or

$$x = -ng \iint dt \, dt \tag{2}$$

The initial conditions associated with the vehicle motion are as follows

$$\begin{aligned} x &= o, \quad \dot{x} = V_i \quad \text{at } t = o \\ x &= x_g, \quad \dot{x} = o \quad \text{at } t = t_1 \end{aligned} \tag{3}$$

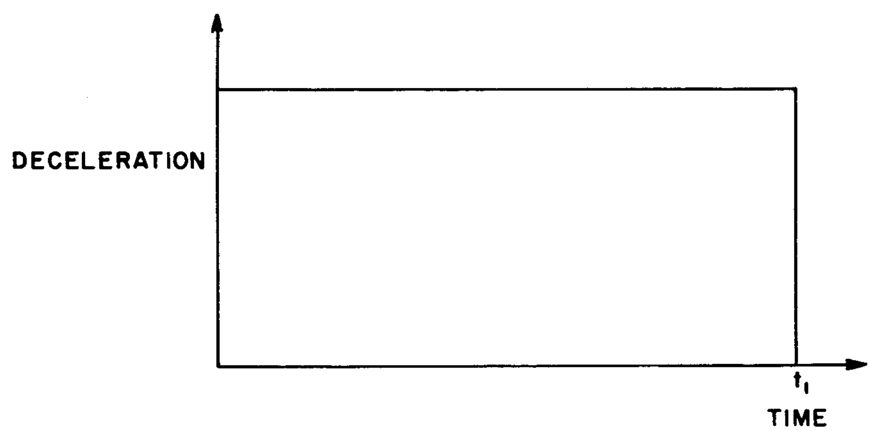
Integration of Equation (2) subject to initial conditions given by Equation (3) gives the following results

$$x_g = (1/2) \; (V_i^2/ng) \tag{4}$$

Equation (4) defines the performance of the deceleration limiter landing system design. For a given touchdown velocity and peak deceleration, the performance is described by the stroke required to absorb the input energy.

FIG. 2

VEHICLE DECELERATION TIME HISTORY WITH A LANDING SYSTEM EMPLOYING A CONSTANT DECELERATION ENERGY ABSORBER



Energy Considerations

A measure of landing system efficiency utilizing the constant deceleration energy absorber (deceleration limiter) can be determined by dividing the total energy absorbed by the system by the weight of the energy dissipating material. Thus,

$$\eta_g = E_a/W_m \tag{5}$$

If it is assumed that the vehicle touchdown kinetic energy is much larger than the change in potential energy of the vehicle associated with the deformation of the landing system, the energy absorbed is simply

$$E_a = (1/2)(W_v/g)(V_i^2) \tag{6}$$

The weight of the energy dissipating material is given directly by Equation (7).

$$W_m = \rho \int A(x)dx \tag{7}$$

For a constant deceleration energy absorber, however, the decelerating force is a constant and equal to the product of the vehicle gross weight and the deceleration factor

$$m\ddot{x} = W_v(\ddot{x}/g) = W_v n \tag{8}$$

If the energy dissipator is designed to operate at a constant stress level, the cross sectional area of the energy dissipating material will be constant and equal to the total decelerating force divided by the working stress. Thus,

$$A = nW_v/\sigma \tag{9}$$

Upon substitution of Equation (9) in (7) and the indicated integration performed, the weight of energy absorbing material is found to be

$$W_m = \rho(nW_v/\sigma)L \tag{10}$$

The deceleration limiter efficiency is then determined by substituting Equations (6) and (10) into Equation (5)

$$\eta_g = (1/2)\,(V_i^2/ng)\,(\sigma/\rho)\,(1/L) \tag{11}$$

Vehicle touchdown velocity can be eliminated from Equation (11) by means of Equation (4). In this manner, the efficiency is given by

$$\eta_g = (\sigma/\rho)\,(x_g/L) \tag{12}$$

As indicated by Equation (12), the landing system efficiency utilizing the deceleration limiter is equal to the product of the material efficiency (stress to density ratio) and the useful stroke to length ratio. Because of the manner in which the energy dissipating material functions, the stroke to length ratio is generally less than unity and for convenience should be treated as a separate variable in this problem.

Weight Efficiency

The weight of the energy absorbing material which is the ideal landing system weight can now be determined simply by dividing the vehicle landing energy by the efficiency of the energy absorber as shown by Equation (5).

$$W_{LS} = E_a/\eta_g \tag{13}$$

If one substitutes the vehicle kinetic energy Equation (6) and the deceleration limiter efficiency Equation (12) into Equation (13) and rearranges terms, the following weight efficiency equation is obtained which relates the landing system weight ratio to material efficiency, stroke to length ratio, and touchdown velocity.

$$W_{LS}/W_v = (\rho/\sigma)\,(L/x_g)\,(V_i^2/2g) \tag{14}$$

This relationship combines the considerations of material efficiency (ρ/σ) structural efficiency (L/x_g) and design conditions $(V_i^2/2g)$ into a single equation which synthesizes the structures/materials/design problem for energy dissipating landing system utilizing solid structural materials.

3. CONSTANT RATE OF DECELERATION ENERGY ABSORBER

The distinguishing characteristic of a constant rate of deceleration energy absorber is its ability to control the time rate of change of deceleration (deceleration onset rate or onset rate). A typical vehicle deceleration time history with a constant onset rate landing system is shown in Figure 3. This figure implies not only that the onset rate is constant but also that the force generated by the energy absorber and resisting vehicle motion increases linearly with time.

As demonstrated in Reference 2, it is possible to shape honeycomb materials so that the force resisting the motion of a mass is developed gradually, and the corresponding onset rate is controlled. Based on the same area-shaping technique for controlling load, it also appears feasible to utilize solid structural materials as onset rate limiters. With this in mind, equations are presented describing the performance and weight of a landing system of the onset limiter type and utilizing a solid structural material as the energy absorber.

Stroke Relationships

The deceleration-time history for an onset limiter is linear with time as shown by Figure 3. The corresponding stroke for a given initial velocity is determined from the double integration of the acceleration-time relationship Equation (1) subject to the initial conditions noted.

$$x = V_i t - \dddot{x}\, g\, t^3/6 \tag{15}$$

and

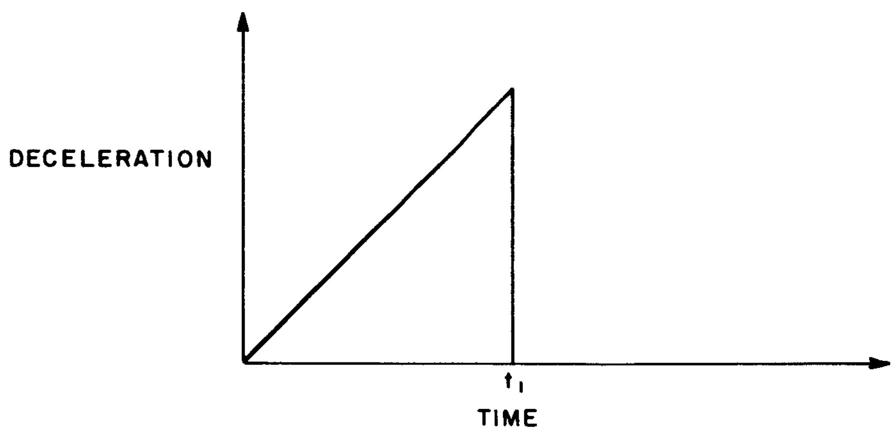
$$\begin{aligned} x &= o \quad \text{at } t = o \\ \dot{x} &= V_i \quad \text{at } t = o \end{aligned} \tag{16}$$

The time (t_1) for the system to reach zero velocity - i.e., absorb all the input energy - is determined by equating the derivative of Equation (15) to zero and solving the resulting equation

$$t_1 = [2V_i/(\dddot{x}\, g)]^{\frac{1}{2}} \tag{17}$$

FIG. 3

VEHICLE DECELERATION TIME HISTORY WITH A LANDING SYSTEM EMPLOYING A CONSTANT RATE OF DECELERATION ENERGY ABSORBER



Equation (17) and (15) can be combined and result in the following relationship between stroke, velocity, and onset rate.

$$x_{\dot{g}} = (2\sqrt{2}/3)\,(V_i)^{3/2}/(\dddot{x}\,g)^{\frac{1}{2}} \tag{18}$$

Since deceleration is linear with time, the deceleration at time "t_1" is

$$n_o = \dddot{x}\,t_1 \tag{19}$$

where n_o is the maximum deceleration in g's. By eliminating "t_1" from Equations (17) and (19) the peak deceleration is found to be

$$n_o = (2\dddot{x}\,V_i/g)^{\frac{1}{2}} \tag{20}$$

Equations (18) and (20) allow the following relationship to be determined among stroke, velocity, and peak deceleration.

$$x_{\dot{g}} = (4/3)\,(V_i^2/n_o g) \tag{21}$$

Equations (20) and (21) define the performance of an onset limiter energy absorber in terms of its maximum deceleration and stroke for a given design input velocity and onset rate.

Energy Considerations

In a manner analogous to that used previously, the efficiency of the onset limiter is determined by dividing the energy absorbed by the material weight or

$$\eta_{\dot{g}} = E_a/W_m \tag{22}$$

The energy absorbing material weight is determined as previously from Equation (7). In order to perform the indicated integration, however, it is first necessary to transform relationships between onset limiter cross sectional area and time into relationships between cross sectional area and stroke.

A linear deceleration vs time history for a vehicle (Fig. 3) implies that the landing system exerts a force on the payload which is also linear with time. Based on this assumption, the relationship between onset limiter cross sectional area and time is, therefore

$$A_{\dot{g}}/A_1 = t/t_1 = \overline{A} \tag{23}$$

where $A_{\dot{g}}$ is the onset limiter cross sectional area as a function of time, A_1 is the area at the time t_1 associated with the end of the stroke, and $\overline{A}$ is a nondimensional area parameter. The equation for stroke as a function of time is given by Equation (15) which is rearranged in the following form

$$x = V_i(t/t_1)t_1 - (\dddot{\overline{x}}\, g/6)\,(t/t_1)^3 t_1^{\,3} \tag{24}$$

Through use of Equations (19) and (20), the parameters t_1 and $\dddot{\overline{x}}$ are eliminated from Equation (24), and the equation put in the following nondimensional form

$$\overline{x} = x/x_{\dot{g}} = (3/2)\,(t/t_1) - (1/2)\,(t/t_1)^3 \tag{25}$$

where the stroke $x_{\dot{g}}$ is given by Equation (21), and $\overline{x}$ is a nondimensional stroke. For given values of the time parameter (t/t_1), Equations (23) and (25) allow the cross sectional area of the onset limiter to be determined as a function of stroke. In this manner the curve of Figure 4 was obtained which shows nondimensional cross sectional area vs stroke for an onset limiter with a stroke to length ratio of unity.

For stroke to length ratios less than unity which may occur depending upon the structural behavior of the onset limiter, it is necessary to make an assumption as to the distribution of material in the region of $L/x_{\dot{g}} > 1.0$. Among other area distributions that could have been chosed for this region the one indicated by the sketch of Figure 5 was utilized in the present analysis since this area distribution corresponds conveniently to an onset limiter in the shape of a cone with a constant wall thickness.

In terms of nondimensional area and stroke, the material weight as given by Equation (7) has the following form

$$W_m = W_{\dot{g}} = \rho\, A_1\, x_{\dot{g}} \int \overline{A}\, d\overline{x} \tag{26}$$

For a stroke to length ratio of unity, a numerical integration of Figure 4 gives a value for the integral of 0.375. The corresponding value for a stroke to length ration less than unity as determined by an integration of the curve

FIG. 4

ONSET LIMITER CROSS SECTIONAL AREA VS. STROKE

$X\dot{g} / L = 1.0$

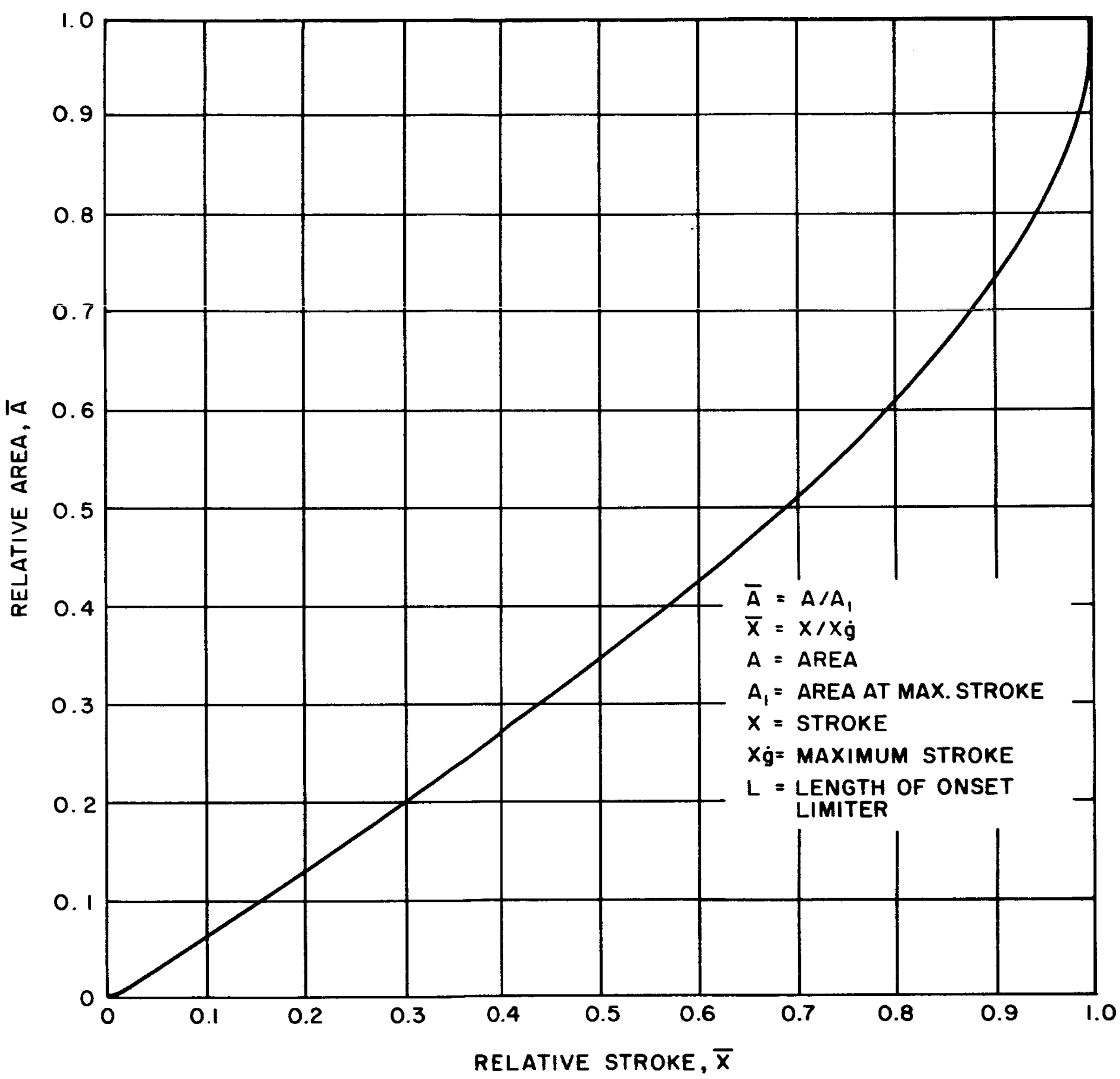
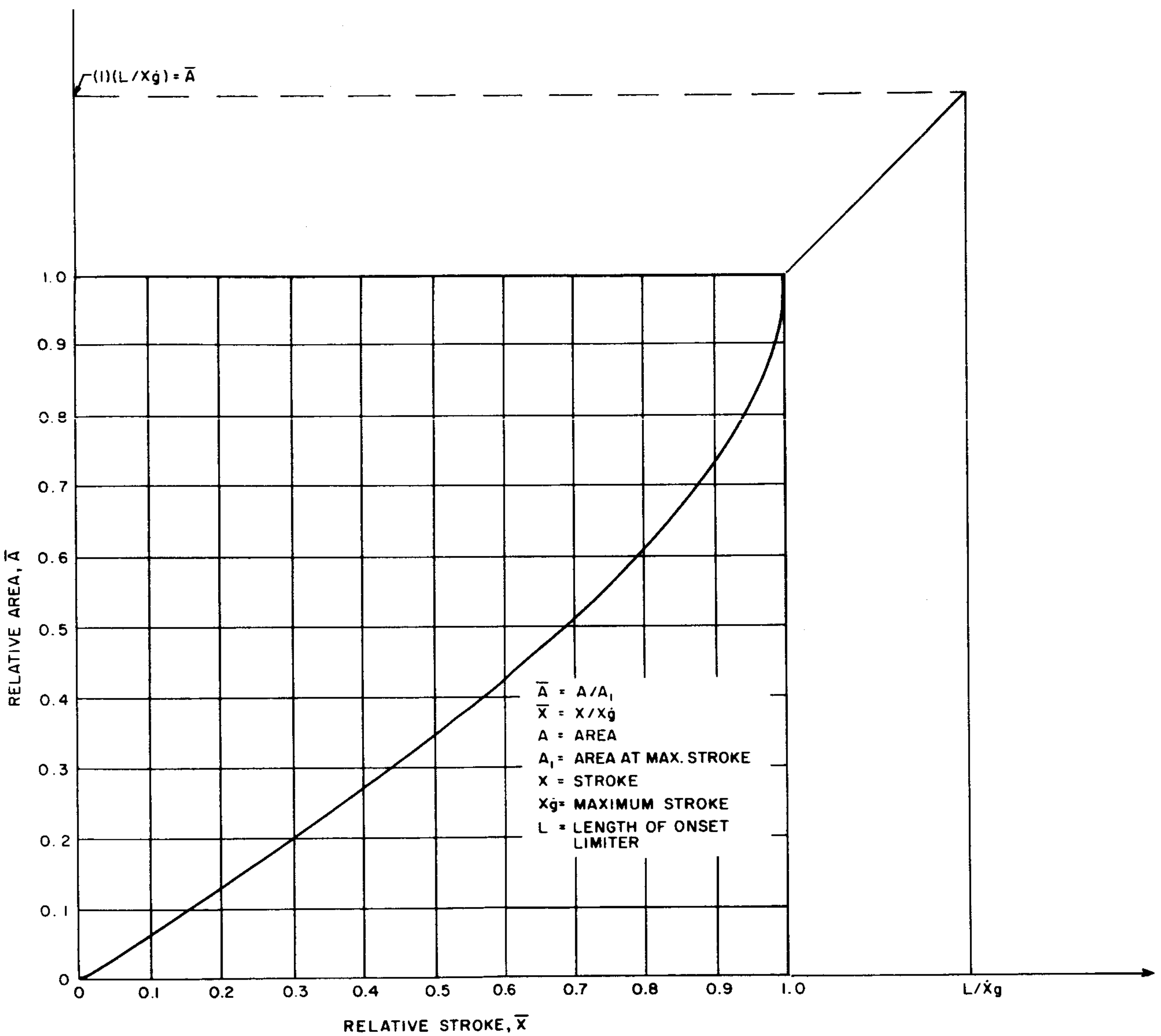


FIG. 5

ONSET LIMITER CROSS SECTIONAL AREA VS STROKE

$\dot{X}g / L \leq 1.0$



of Figure 5 is

$$\int \bar{A}\, d\bar{x} = (1/2)\,(L/x_{\dot{g}})^2 - .125 \tag{27}$$

The material cross sectional area which corresponds to maximum stroke is determined by dividing the peak decelerating force in the material by the constant operating stress as given by Equation (9). Upon substitution of Equations (9) and (27) into (26), the following equation is obtained for the energy absorbing material weight

$$W_m = (\rho/\sigma)\, n_o W_v x_{\dot{g}}\, [(1/2)\,(L/x_{\dot{g}})^2 - .125] \tag{28}$$

As previously, it is assumed that the vehicle potential energy associated with deformation of the energy dissipator is negligible compared with the touchdown kinetic energy of the vehicle; hence, the total energy absorbed by the onset limiter is assumed equal to the vehicle touchdown kinetic energy. Based on this assumption, the onset limiter efficiency is found from Equation (28)

$$\eta_{\dot{g}} = (1/2)\,(\sigma/\rho)\,(V_i^2/n_o g)\,(x_{\dot{g}}\,[(1/2)\,(L/x_{\dot{g}})^2 - .125])^{-1} \tag{29}$$

A simplified equation for onset limiter efficiency is found by substituting Equation (21) into (29) and is given by

$$\eta_{\dot{g}} = (\sigma/\rho)\,(3/8)\,[(1/2)\,(L/x_{\dot{g}})^2 - .125]^{-1} \tag{30}$$

As shown by Equation (30) the onset limiter efficiency is a function of the material efficiency and the square of the stroke to length ratio.

Weight Efficiency

As outlined in Section 2, the landing system weight is simply the vehicle kinetic energy divided by the efficiency of the energy dissipator, Equation (13). The substitution of vehicle kinetic energy and onset limiter efficiency Equation (30) into Equation (13) allows the following weight ratio to be determined.

$$W_{LS}/W_v = (\rho/\sigma)\,[(4/3)\,(L/x_{\dot{g}})^2 - .333]\,(V_i^2/2g) \tag{31}$$

4. COMPOSITE ENERGY ABSORBER

Since the fragility limitations of most vehicle payloads impose limits on both the onset rate as well as the peak deceleration, a landing system which combines the desirable characteristics of both the onset limiter and the deceleration limiter is of interest. A typical deceleration time history for a vehicle using a composite energy absorber which includes both an onset and deceleration limiter is shown in Figure 1. As indicated, the behavior of this idealized landing system results in an increase in vehicle deceleration at a constant onset rate until a maximum deceleration is reached; subsequently, the vehicle experiences a constant deceleration at zero onset rate. For convenience, the deceleration time history for the composite system has been divided into an onset limiter region and deceleration limiter region.

In the following, equations are presented which describe the performance and weight of the composite energy absorbing system. In addition, methods are given for synthesizing the performance requirements, weight, and design parameters into a single graphical presentation which allows landing system weight to be determined for all possible combinations of design conditions.

Stroke Relationships

The stroke of the composite system can be determined from equations describing the behavior of its component elements as presented in Section 2 and 3. With this approach, the form of the stroke equation is

$$x_c = x_{\dot{g}} + x_g \tag{32}$$

An equation for the stroke associated with the onset limiting portion of the deceleration cycle is found by combining Equations (15) and (19). Thus,

$$x_{\dot{g}} = (V_i\, n_o/\dddot{\bar{x}}) - g\, n_o{}^3/(6\,\dddot{\bar{x}}^2) \tag{33}$$

It is convenient to introduce a nondimensional parameter, λ, which is defined as follows

$$\lambda = n_o{}^2 g/(V_i \dddot{\bar{x}}) \tag{34}$$

Equation (33) may now be expressed as a function of λ and has the following form

$$x_g = [V_i^2/(n_o g)] (\lambda - \lambda^2/6) \tag{35}$$

Equation (35) defines the stroke of the onset limiting position of the composite energy absorbing system. It is noted that λ has a value of 2 for a pure onset limiter energy absorber which can be verified by substituting Equation (20) into (34). The corresponding value for the complete λ function in Equation (35) is 4/3; hence, Equation (35) agrees with Equation (21) for the limiting case when all the landing energy is absorbed as an onset limiter dissipator.

For a deceleration limiter, the onset rate is infinite, and the corresponding value of λ from Equation (34) is zero. For this value of λ, the stroke contribution of the onset limiter from Equation (35) is also zero. Thus Equation (35) also satisfies the limiting case where all energy is absorbed via a deceleration limiter.

The stroke associated with the deceleration limiting portion of the composite system can be determined from Equation (4) providing the intial velocity is suitably defined. For completeness, Equation (4) is rewritten in the amended form shown below.

$$x_g = \dot{x}_1^2/(2n_o g) \tag{36}$$

where $\dot{x}_1$ is the vehicle velocity at the beginning of the deceleration limiter stroke. The incipient velocity at the beginning of the deceleration limiter stroke, or conversely the velocity at the end of onset limiter stroke, is determined by differentiating Equation (15) with respect to time and substituting in the resulting equation the value for t_1 from Equation (19). In this manner, $\dot{x}_1$ is found to be

$$\dot{x}_1 = V_i - n_o^2 g/(2\dddot{x}) \tag{37}$$

If one substitutes Equation (37) into (36) and introduces the λ parameter (Eq. 34), the following equation is obtained for the constant deceleration portion of the total composite stroke.

$$x_g = [V_i^2/(2n_o g)]\,(1 - \lambda + \lambda^2/4) \tag{38}$$

Again, it is noted that the above equation reduces to the proper form for the limiting conditions of pure deceleration limiter ($\lambda = 0$, $x_g = V_i^2/(2n_o g)$ and pure onset limiter ($\lambda = 0$, $x_g = 0$).

From Equation (32) the total stroke of the composite system is the sum of Equations (33) and (38) which gives

$$x_c = [V_i^2/(n_o g)]\,(1/2 + \lambda/2 - \lambda^2/24) \tag{39}$$

Equation (39) defines the total stroke of the energy absorbing system as a function of the touchdown velocity, peak deceleration, and the design parameter λ.

Energy Considerations

The energy absorbing efficiency of a deceleration and onset limiter have been determined in Sections 2 and 3 based on the assumption that these elements each absorb the total landing kinetic energy. For a composite system such as indicated by Figure 1, each element, however, will absorb only a fraction of the total kinetic energy. Because of this partitioning of energy, it is necessary to re-examine the efficiency of these components when integrated into a composite system.

The total landing system kinetic energy is considered to be apportioned to each element of the energy absorber as indicated

$$KE_T = KE_{\dot{g}} + KE_g \tag{40}$$

By definition, the kinetic energy absorbed by the onset limiter is

$$KE_{\dot{g}} = (1/2)\,m\,(V_i^2 - \dot{x}_1^2)$$

$$= KE_T\,(1 - \dot{x}_1^2/V_i^2) \tag{41}$$

where $KE_T = (1/2)\,m\,V_i^2$

From Equation (41), the fraction of the total energy absorbed by the onset limiter is

$$KE_{\dot{g}}/KE_T = 1 - (\dot{x}_1/V_i)^2 \tag{42}$$

The velocity ratio $\dot{x}_1/V_i$ may be evaluated from Equations (37) and (34)

$$\begin{aligned} \dot{x}_1/V_i &= 1 - (1/2)\,(n_o{}^2 g/\dddot{\bar{x}}\,V_i) \\ &= 1 - (1/2)\,\lambda \end{aligned} \tag{43}$$

Substitution of Equation (43) in (42) gives

$$KE_{\dot{g}}/KE_T = \lambda - \lambda^2/4 \tag{44}$$

From Equation (40), the fraction of the total kinetic energy absorbed by the deceleration limiter is

$$KE_g/KE_T = 1 - KE_{\dot{g}}/KE_T \tag{45}$$

Substitution of Equation (44) in (45) gives

$$KE_g/KE_T = 1 - \lambda + \lambda^2/4 \tag{46}$$

Equations (44) and (46) define the portion of total energy absorbed by the onset and deceleration limiter in terms of the design parameter λ. These equations may now be used to calculate the efficiencies of the composite components. The development follows that for the deceleration and onset limiter efficiency given in Sections 2 and 3.

As shown by Equation (5), the efficiency of the deceleration limiter is obtained by dividing the energy absorbed by the weight of the energy dissipating material. For this application, the energy absorbed is obtained from Equation (46); however, the equation for the weight of the energy dissipating material will be the same as given by Equation (10). From Equations (5), (46), and (10), the efficiency is

$$\eta_g = KE_T\,(1 - \lambda + \lambda^2/4)\,(\sigma/\rho)\,(n_o W_v L_g)^{-1} \tag{47}$$

Introducing in Equation (47) the equivalent form for the total kinetic energy of $(1/2)\,(W_v/g)\,V_i{}^2$ and rearranging terms, one obtains

$$\eta_g = (\sigma/\rho)\,[V_i{}^2/(2n_o g)]\,(1 - \lambda + \lambda^2/4)\,(\ L_g)^{-1} \tag{48}$$

Substitution of Equation (38) in (48) gives finally,

$$\eta_g = (\sigma/\rho)(x_g/L_g) \tag{49}$$

As indicated, the efficiency of the deceleration limiter for the composite system is the product of the material efficiency and the stroke to length ratio. A comparison of this result with Equation (12) reveals that the efficiency is the same as that for the deceleration limiter acting as the sole energy absorber.

In a manner identical to that for the deceleration limiter, the efficiency of the onset limiter is found by dividing the energy absorbed by the weight of the energy absorbing material (See Eq. (22)). For the onset limiter acting in a composite system, the energy absorbed is obtained from Equation (44). Additional considerations are necessary to determine the weight of the energy absorbing material and are presented below.

As in Section 3, it is assumed that the distribution of material cross sectional area for the onset limiter is given by Figure 5. From Equation (26) and Figure 5, the material weight is

$$W_m = \rho A_1 x_g \left\{ \int_0^{1.0} \bar{A}\, d\bar{x} + \int_{1.0}^{L_{\dot{g}}/x_{\dot{g}}} \bar{A}\, d\bar{x} \right\} \tag{50}$$

The second integral given by Equation (50) which represents the weight penalty associated with designs whose stroke-to-length ratio is less than unity can be determined by graphical integration and is

$$\int_{1.0}^{L_{\dot{g}}/x_{\dot{g}}} \bar{A}\, d\bar{x} = (1/2)\left[(L_{\dot{g}}/x_{\dot{g}})^2 - 1\right] \tag{51}$$

The first integral given by Equation (50) represents the volume of material associated with the working stroke of the energy absorber. This integral is evaluated by noting that the work done by the energy absorber (as determined by the integral of the force developed by the onset limiter times the stroke) must equal the kinetic energy absorbed. The internal work done by the onset limiter is

$$Wi_{\dot{g}} = \int F\, dx = \sigma A_1 x_{\dot{g}} \int_0^{1.0} \bar{A}\, d\bar{x} \tag{52}$$

Equating this internal work to the external energy absorbed (Eq. 44) and rearranging terms, one obtains

$$\int_0^{1.0} \overline{A}\, d\overline{x} = (1/2)\,(W_v/g)\,(V_i)^2\,(\sigma\, A_1\, x_{\dot{g}})^{-1} \tag{53}$$

Further simplification of Equation (53) results by substituting Equation (35) in (53) and noting that the cross sectional area corresponding to maximum stroke is

$$A_1 = n_O\, W_V/\sigma \tag{54}$$

The resulting equation for nondimensional volume is

$$\int_0^{1.0} \overline{A}\, d\overline{x} = (1/2)\,(\lambda - \lambda^2/4)/(\lambda - \lambda^2/6) \tag{55}$$

It is noted that for an onset limiter as the sole energy absorber ($\lambda = 2.0$) the value of the integral is 0.375 which agrees with results presented in Section 3. For a composite energy absorber, λ will be less than two,and it follows from Equation (55) that the corresponding value for the integral will lie between 0.375 and 0.50.

The material weight for the onset limiter as found by substituting Equations (51) and (55) in (50) and introduction Equations (35) and (54) is

$$W_m = (\rho/\sigma)\,(W_v/g)\,(V_i^2)\,[\lambda - \lambda^2/6]\left\{(1/2)\,(\lambda - \lambda^2/4)/(\lambda - \lambda^2/6) + (1/2)\,[(L_{\dot{g}}/x_{\dot{g}})^2 - 1]\right\} \tag{56}$$

The efficiency is then found by dividing the energy absorbed (as obtained from Eq. (44)) by the material weight (Eq. (56)) which gives

$$\eta_{\dot{g}} = (\sigma/\rho)_{\dot{g}} / \left\{1 + [(L_{\dot{g}}/x_{\dot{g}})^2 - 1]\,(\lambda - \lambda^2/6)/(\lambda - \lambda^2/4)\right\} \tag{57}$$

Equation (57) expresses the onset limiter efficiency for a composite energy absorber as a function of the material efficiency, stroke-to-length ratio, anddesign parameter λ.

In contrast to the results for the deceleration limiter, a comparison of Equations (57) and (30) reveals that the onset limiter efficiency in a composite system is not the same as that for an onset limiter used as the sole energy absorber. It can be determined from Equation (57) and (30) that the efficiency of the onset limiter in a composite system (i.e., $\lambda < 2.0$) is greater than the efficiency of the onset limiter as a sole energy absorber. It is noted, however, that Equation (57) reduces to Equation (30) for the limiting case of an onset limiter energy absorber only ($\lambda = 2.0$).

Weight Efficiency

The weight of the landing system for the composite energy absorber can be expressed as the sum of the weights of its elements as shown

$$W_{LS} = W_{LS_{\dot{g}}} + W_{LS_g} \tag{58}$$

Equation (58) can also be expressed in terms of the kinetic energy absorbed by each element of the composite system and their associated efficiencies as follows

$$\begin{aligned} W_{LS} &= KE_{\dot{g}}/\eta_{\dot{g}} + KE_g/\eta_g \\ &= (KE_T/\eta_g)\,[(\eta_g/\eta_{\dot{g}})\,(KE_{\dot{g}}/KE_T) + KE_g/KE_T] \end{aligned} \tag{59}$$

By substituting the kinetic energy ratios given by Equations (44) and (46) in (59), one obtains

$$W_{LS} = (KE_T/\eta_g)\,[(\eta_g/\eta_{\dot{g}})\,(\lambda - \lambda^2/4) + 1 - \lambda + \lambda^2/4] \tag{60}$$

A comparison of Equations (49) and (57) reveals that for the same material efficiency and stroke to length ratio, where the stroke to length ratio is less than one, the deceleration limiter is a more efficient energy absorber than the onset limiter. The ratio of total vehicle kinetic energy to deceleration limiter efficiency (see Eq. (60)), therefore, can be interpreted as the optimum landing system weight and has a minimum value when the stroke to length ratio is one (see Eq. (49)). Based on these considerations, Equation (60) may be rewritten in the following form

$$W_{LS}/\overline{W}_{LS} = (L_g/x_g)\,[(\eta_g/\eta_{\dot{g}})\,(\lambda - \lambda^2/4) + 1 - \lambda + \lambda^2/4] \tag{61}$$

where $\overline{W}_{LS}$ is the minimum landing system weight and equals the total kinetic energy divided by the material efficiency of the deceleration limiter. From Equations (49) and (57), the efficiency ratio appearing in Equation (61) is given by

$$\eta_g/\eta_{\dot{g}} = (x_g/L_g)\left\{1+[L_{\dot{g}}/x_{\dot{g}})^2-1](\lambda-\lambda^2/6)/(\lambda-\lambda^2/4)\right\}(\sigma/\rho)_g/(\sigma/\rho)_{\dot{g}} \tag{62}$$

The ratio of landing system weight to minimum weight (Eq. (61)) gives the weight penalty associated with 1) the additional restrictions of a limited onset rate as imposed by the composite energy absorber and 2) a stroke to length ratio less than unity for the components.

The ratio of landing system weight to vehicle gross weight can now be determined from the product of the ratio of minimum landing system weight to vehicle weight and the weight penalty ratio as shown

$$W_{LS}/W_v = (\overline{W}_{LS}/W_v)(W_{LS}/\overline{W}_{LS}) \tag{63}$$

The minimum landing system weight to vehicle gross weight ratio $(\overline{W}_{LS}/W_v)$, as previously noted, is based on absorbing the total kinetic energy via a deceleration limiter whose stroke to length ratio is unity. From Equation (14) this ratio is

$$\overline{W}_{LS}/W_v = (\rho/\sigma)_g (V_i^2/2g) \tag{64}$$

Substitution of Equations (61) and (64) in (63) gives

$$W_{LS}/W_v = (\rho/\sigma)_g(L/x)_g[(\eta_g/\eta_{\dot{g}})(\lambda-\lambda^2/4) + 1-\lambda+\lambda^2/4](V_i^2/2g) \tag{65}$$

This equation represents the synthesis of the material efficiency, structural efficiency, and design conditions for the landing system using a composite energy absorber. Similar equations have previously been presented for the deceleration limiter (Eq. (14)) and the onset limiter (Eq. (31)).

Design Synthesis

A synthesis of the weight, performance, and design parameters developed in previous subsections is presented below. In this manner, the design indices which determine landing system weight are defined. The results are presented as a design chart which gives the optimum landing system to vehicle weight ratio for any given value of the design index.

The equations for landing system performance and weight have been expressed as a function of the design parameter λ. As shown by Equation (34), this parameter is a function of peak deceleration factor, touchdown velocity, and onset rate. It is desirable to introduce a more general design parameter which includes landing system stroke in addition to the parameters noted and is defined by

$$\beta = n_o V_i / (x_c \bar{\dddot{x}}) \tag{66}$$

From Equation (39), the parameter β is related to λ as follows

$$\beta = \left\{ 1/2 - \lambda/24 + 1/(2\lambda) \right\}^{-1} \tag{67}$$

As noted previously, the range for λ is from zero (deceleration limiter only) to two (onset limiter only), and the corresponding range for β as determined by Equation (67) is zero (deceleration limiter only) to 1.5 (onset limiter only). Equation (67) is essentially a nondimensional form of the composite stroke equation (Eq.(39)).

From Equations (63) and (64) the landing system weight ratio can be written as follows

$$(g)(\sigma/\rho)_g (W_{LS}/W_v) = (V_i^2/2)(W_{LS}/\overline{W}_{LS}) \tag{68}$$

If it is assumed that the material efficiencies for the onset limiter and deceleration limiter are equal, the weight penalty ratio as determined by the substitution of Equation (62) in (61) is given by

$$W_{LS}/\overline{W}_{LS} = (L_g/x_g)\,[(x_g/L_g)\left\{1 + [(L_{\dot{g}}/x_{\dot{g}})^2 - 1](\lambda - \lambda^2/6)/(\lambda - \lambda^2/4)\right\}(\lambda - \lambda^2/4)$$
$$+ 1 - \lambda + \lambda^2/4] \tag{69}$$

A further simplification of Equation (69) results if it is assumed that the stroke to length ratio are equal for the onset and deceleration limiter (note that for this condition the stroke to length ratio for the composite system will be the same as its components). Thus, Equation (69) reduces to

$$W_{LS}/\overline{W}_{LS} = (L/x_c)^2 (\lambda - \lambda^2/6) + (L/x_c)(1-\lambda+\lambda^2/4) - \lambda^2/12 \qquad (70)$$

Equations (67) and (70) allow the weight penalty ratio to be determined as a function of the design parameter β. Figure 6 presents a curve of this relationship for a stroke-to-length ratio of 0.8. As shown by this figure, the weight penalty ratio for a deceleration limiter only ($\beta = 0$) is 1.25 and this value increases to a maximum of 1.75 for an onset limiter only ($\beta = 1.5$).

To facilitate a graphical presentation of the landing system weight ratio given by Equation (68) it is desirable to introduce the design index θ as defined below

$$\theta = x_c \overline{\dddot{x}} / n_o \qquad (71)$$

From Equation (66), the design parameter β can now be expressed as a function of the design index and the vehicle touchdown velocity. Thus

$$\beta = V_i / \theta \qquad (72)$$

For a fixed value of the design index θ, the design parameter β and the associated value of the weight penalty ratio (see Fig.6) becomes a function of velocity. For a given θ, therefore, the right-hand side of Equation (68) becomes a function of velocity only. Hence, Equation (68) with Equation (72) and Figure 6 allow the determination of the landing system weight ratio as a function of touchdown velocity for any value of the design index θ.

Figure 7 shows curves of landing system weight ratio versus vehicle design parameters. The abscissa is vehicle touchdown velocity, and the ordinate is the product of the planetary acceleration due to gravity, material efficiency, and the landing system weight ratio. For a given material of construction and gravity acceleration, Curve A gives the landing system weight ratio as a function of velocity for an ideal deceleration limiter system--i.e., a system with a stroke to length ratio of one. This curve was obtained from Equation (68) with a weight penalty ratio of unity.

FIG. 6

WEIGHT PENALTY RATIO VS. VEHICLE DESIGN PARAMETER

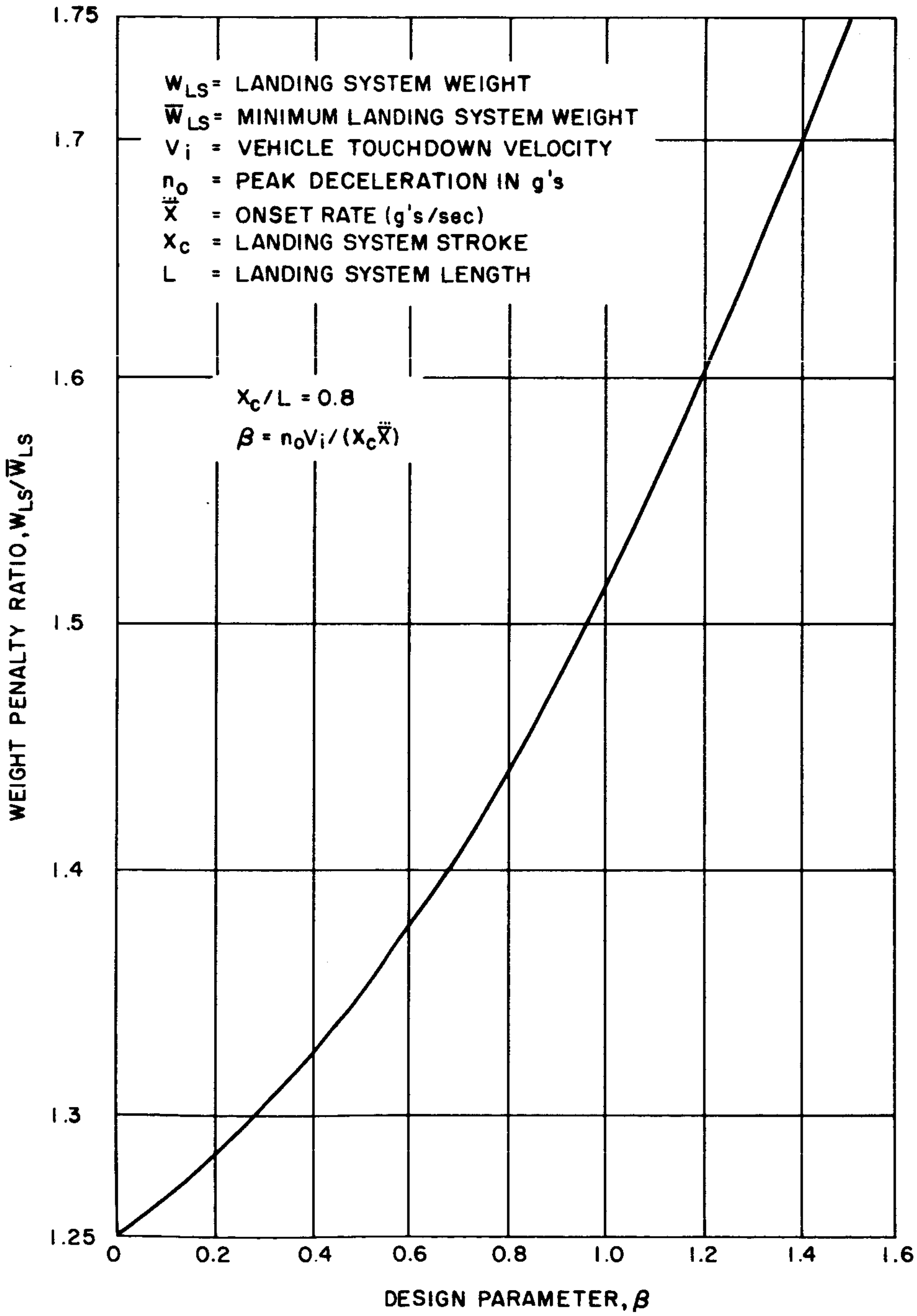
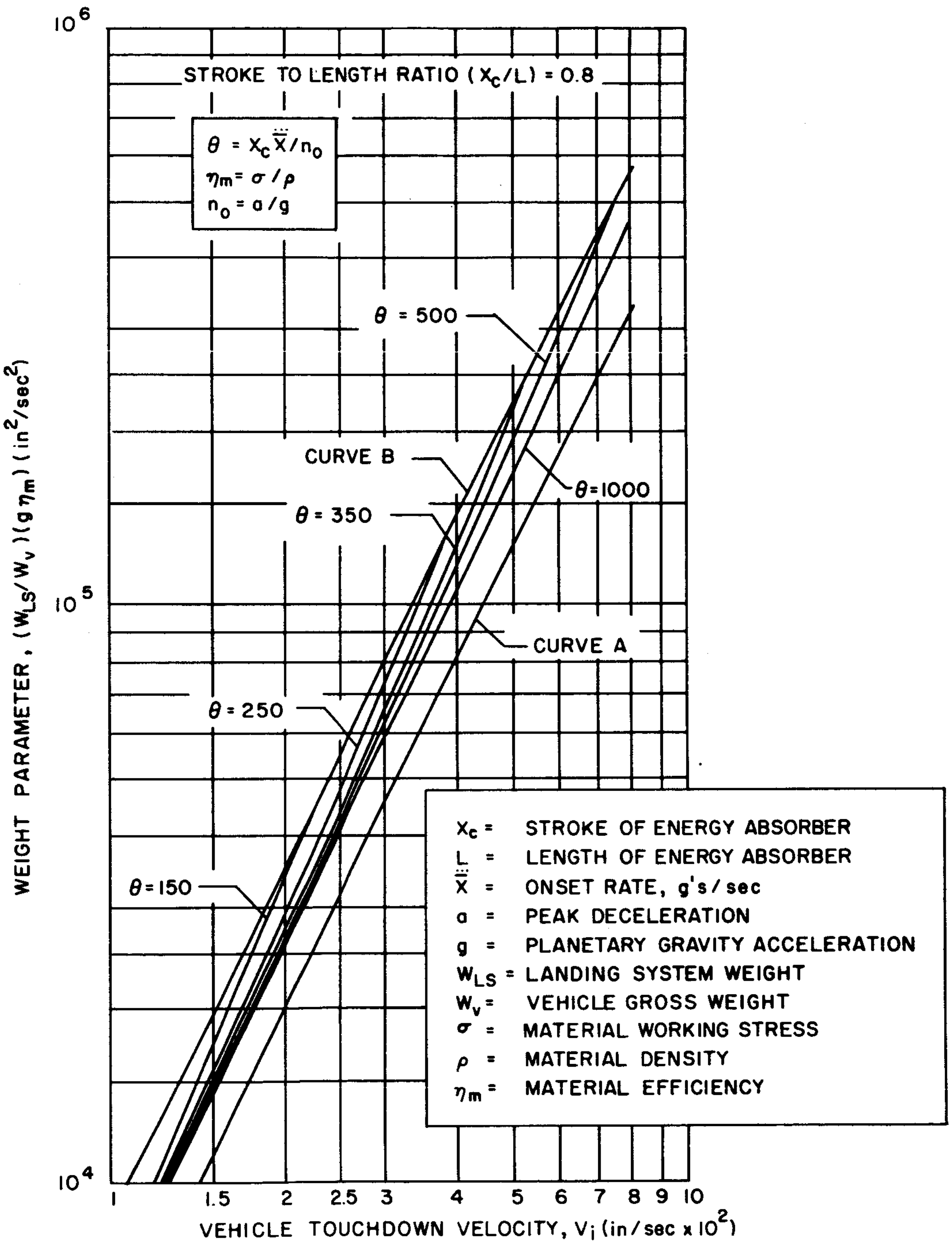


FIG. 7

OPTIMUM LANDING SYSTEM WEIGHT RATIO VERSUS VEHICLE DESIGN PARAMETERS



Curve B shows the variation of landing system weight ratio and velocity for an onset limiter system only. For a stroke to length ratio of 0.8, the weight penalty ratio for this system is 1.75 as noted above. Accordingly, Curve B is a plot of Equation (68) with the weight penalty ratio equal to 1.75. The θ curves give landing system weight vs velocity associated with a composite energy absorbing element. As indicated above, these curves were obtained by 1) calculating values of β as a function of velocity using Equation (72) and 2) substituting in Equation (68) the value of the weight penalty ratio corresponding to β as determined from Figure 6.

Figure 7 is the end result of the landing system design synthesis. Through use of curves such as presented in Figure 7 the landing system weight for an ideal energy absorber using a solid material can readily be determined once the design and performance parameters are specified. As shown the appropriate design index which fixes the landing system weight is a simple combination of the parameters stroke, onset rate, and peak deceleration.

5. DISCUSSION

Presented below is a review of assumptions and results associated with the estimation of the weight and performance of an optimum landing system design. Other possible analytical approaches to the problem are also discussed. Finally, the need for additional theoretical investigations is suggested.

In order to simplify the analysis, the potential energy associated with the change in position of the vehicle mass due to deformation of the landing system was neglected. Since the stroke of a landing system using a solid structural material as an energy absorber is small, the change in vehicle potential energy due to stroke is also small. It can be shown that the ratio of vehicle potential energy associated with the landing system stroke to the touchdown kinetic energy is approximately equal to the reciprocal of the peak deceleration factor. Therefore, for a peak deceleration exceeding 10g, the potential energy is less than ten percent of the kinetic energy.

As noted in the introduction, a desirable characteristic of a landing system is that it attenuate vehicle motions with a minimum of rebound. For a solid structural material energy absorber, this implies that the working stroke is associated with inelastic material deformation. Since landing system efficiency increases as the material efficiency increases, the most efficient system for a given material will correspond to a design which has the highest working stress. In general, the maximum attainable material efficiency will be equal to the compressive yield stress to density ratio.

It is apparent from Figure 7 that the minimum weight landing system is one wherein energy dissipation is of the deceleration limiter type (Curve A). The highest landing system weight (Curve B) corresponds to an energy absorption system of the onset limiter type. As can be determined by a comparison of Equations (49) and (57), the relatively superior weight performance of the deceleration limiter is due to its higher efficiency for stroke to length ratios less than one. For a theoretically ideal system wherein the stroke to length ratio is unity, however, the efficiencies of both systems would be equal and the corresponding landing system weights would also be equal.

Although the pure deceleration limiter landing system is desirable from weight considerations, the associated onset rate would probably be intolerable for most vehicle payloads. A landing system design of the onset limiter type, however, might exceed the tolerable deceleration limits of the payload. The composite system which has a controlled onset rate and peak deceleration is the most desirable for landing system application. From considerations presented above, it is apparent that a minimum weight composite system will correspond to a design wherein the energy absorbed by the onset limiter is small compared to that absorbed by the deceleration limiter.

With the aid of curves such as shown in Figure 7, the landing system weight ratio can readily be determined for any given set of design indices. For example, the peak deceleration and onset rate for a given vehicle are established by payload fragility. The landing system stroke (size of the energy absorber) could conceivably be specified by the storage space available between the payload and the booster. From such considerations, a value of the θ-index is determined. For any given touchdown velocity, the corresponding weight ratio-material efficiency-gravitational acceleration parameter can then be determined from curves such as Figure 7. As previously noted, the maximum material efficiency parameter for solid materials is the compressive yield stress to material density ratio.

Hence, once the energy absorbing material and the acceleration due to gravity is known, the landing system to vehicle gross weight ratio is readily determined. This weight ratio is the optimum that can be achieved for the design conditions specified since it is based on an ideal energy absorbing material and also does not reflect the additional weight required to incorporate an energy absorbing material into a working system on the vehicle. The simplicity by which the design indices significant to landing system design are presented in Figure 7 facilitates the determination of landing system weight penalties which may be associated with a change in any of a given set of parameters.

Although the curves of Figure 7 are based on the assumption that both the onset limiter and deceleration limiter have the same material efficiency and stroke to length ratio, the analytical development given in Section 4 is not dependent on such restrictions. For example, curves similar to Figure 7 could have been prepared for a landing system in which both the onset limiter and deceleration limiter had different but prescribed values of material efficiency and stroke to length ratio.

The weight and performance of the composite energy absorber presented in this report was based on the assumption that an ideal landing system design would produce a vehicle deceleration time history shown in Figure 1--viz., the deceleration increases linearly with time to a maximum value and then remains constant. Although not included in this report, a similar investigation was also made for a system wherein the vehicle deceleration increased linearly with stroke to a maximum value and thereafter remained constant until all the landing energy was dissipated. In general, there is very little difference between the weight and performance of the landing systems corresponding to the two different vehicle deceleration characteristics. For the same design conditions--i.e., peak deceleration, onset rate, and touchdown velocity--it was determined that both the weight of the landing system and the stroke was a little smaller (on the order of a few per cent) for the deceleration vs time as compared with the deceleration vs stroke vehicle deceleration model. It is noted that for the same peak deceleration, the onset limiter design based on linear deceleration vs stroke considerations absorbs a larger fraction of the total vehicle kinetic energy than the onset limiter of this report. For both decelerations signatures, the energy absorbing efficiency of the deceleration limiter is the same, and in both cases, the deceleration limiter is more efficient than the onset limiter.

The results of this study as represented by the design curves of Figure 7 for estimating the optimum landing system weight are presently applicable only for solid material energy absorbers. Future work should be directed towards generalizing the analytical techniques developed such that the weight of various systems--e.g., braking rockets, gas-bags, solid materials--could be compared on the basis of the same or similar design indices.

6. CONCLUSIONS AND RECOMMENDATIONS

1. An ideal landing system design will control both the onset rate and peak deceleration of the vehicle payload.

2. By mean of analytical techniques presented in this report, the optimum weight of a composite landing system composed of onset and deceleration limiting elements and using solid structural materials can be related to vehicle touchdown velocity and a simple design index. With this information the optimum landing system weight can be determined once the design parameters of peak deceleration, onset rate, and stroke are known.

3. For solid material energy absorbers with the same material efficiency and stroke to length ratio, the absorption of impact energy via an onset limiting device will require a greater landing system weight than for the deceleration limiter.

4. For the same design conditions, landing system weight and performance are essentially the same for vehicle deceleration signatures which 1) increase linearly with time to a maximum value and then remain constant until vehicle motion ceases or 2) increases linearly with stroke to a maximum value and remain constant until vehicle motion is arrested.

5. Future work should be directed towards generalizing the analytical techniques developed in this report such that the weight of various systems (e.g., braking rockets, gas-bags, solid materials) could be compared on the basis of the same or similar design indices.

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